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Investigating Quality of Undergraduate Mathematics Lectures

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The notion of quality in undergraduate mathematics lectures is examined by using theoretical notions and research results from the literature and empirical data from a case study on lecturing on limits of functions. A systemic triangular model is found to catch critical quality aspects of a mathematics lecture, consisting of mathematical exposition, teacher immediacy, and general quality criteria for mathematics teaching. Mathematical exposition involves the dynamic interplay of mathematical content, mathematical process, and institutionalisation. The discussion is a contribution to an increased pedagogical awareness in undergraduate mathematics teaching.

The lecture has a long history as a teaching format at universities. Depending on countries and traditions it goes along with tutoring, seminars, classes, small group work (including computer laboratories), and home assignments as examples of traditional additional teaching offered to students. Since the lecture is still one of the major formats used in undergraduate mathematics education, the frequent doubts about its value (Bligh, 1972; Fritze & Nordkvelle, 2003; Holton, 2001) have inspired investigations into the possible characteristics of a *quality lecture* for beginning university students. Thus, in this paper the notion of quality in undergraduate mathematics lectures will be examined by using theoretical notions and research results from the literature and new empirical data from a case study on one lecture about limits of functions. The resulting analyses will be used as a basis to reflect on the following research question: *What are the main factors that account for quality in an undergraduate mathematics lecture*? The point of view taken here is that of an observer-what is actually happening in the lecture hall? The key aspect of students' learning and appreciation of a lecture, without which discussions of quality will remain on a theoretical level, and how that aspect relates to the findings presented in this paper, is the focus of an ongoing follow up study. It has been documented that students and lecturers may have different views on these issues (Anthony, Hubbard, & Swedosh, 2000, pp. 250-251).

With a *lecture* I will here refer to a time scheduled oral presentation on a pre-announced topic to a large group of people, where the speaker (mostly alone) is overlooking the "crowd" from a podium position, and the people in the "crowd" are sitting (close) together in lines of chairs facing the speaker. It is a social and situational setting in which both general and specific frames come in play, related both to the scientific and educational faces of a university. A lecture in a course, which will be the focus in this paper, is one of a series of lectures constituting the course, along with textbooks and other teaching activities.

In a study on lectures from a systems theory perspective, Fritze and Nordkvelle (2003) identify three different functions of a lecture: "as exposing two social systems: science and education...it symbolically represents science as well as it represents *the educational system*" (p. 332). The lecturer thus demonstrates scientific truth by ways of argumentation and reflection, and secondly takes educational decisions in order to make this scientific content accessible to students. In addition, since students regard lectures in a course as "a part of a socialization scheme," the lecture takes on a third function of an "organization activity" (ibid.).

Different styles of lectures have been identified by Saroyan and Snell (1997) — the *content-driven*, *context-driven*, and *pedagogy-driven* lecture. In mathematics, what is often called a traditional lecture is content-driven, a one-way communication using the *definition* — *theorem* — *proof* format (DTP format) (Weber, 2004), focussed on presentation in the DTP order of within mathematics content matter only.

In the next section the notion of quality teaching is briefly discussed. Some previous research on undergraduate mathematics lecturing is then presented, forming part of the conceptual framework used for addressing the research question as stated above. The methodology and results of the case study are described, followed by a discussion and conclusions leading to a tentative model of factors accounting for quality in a lecture.

Quality Teaching

The objective of teaching is students' learning and the fact that the term *quality teaching* is used in the literature (see below for references) presupposes a connection between how teaching is done and the learning in students that takes place. Indeed, to understand this connection is one of the main aims of educational research. The level or quality of student learning can be expressed in terms of knowledge characteristics, such as the commonly used notions skills, understanding, instrumental and relational knowledge, mathematical competencies, mathematical proficiency, or levels of the SOLO taxonomy. This makes the idea of quality in teaching relative to the aimed target knowledge of the education. In addition, adopting a constructivist approach to learning, a specific teaching intervention in a class may well lead to different constructions of knowledge in different students in the class, depending on their previous knowledge and the web of contextual factors.

Biggs (2003, p. 75) lists four teaching/learning contexts that from the literature seem to support quality learning:

1. A well-structured knowledge base.
2. An appropriate motivational context.
3. Learner activity, including interaction with others.
4. Self-monitoring.

These contexts are in resonance with the general criteria for quality in mathematics teaching as proposed in Blum (2004), where three strands are seen as critical, based on support from empirical research:

- *Demanding orchestration of the teaching of mathematical subject matter* (competence oriented, creating opportunities to acquire these, and making connections),
- *Cognitive activation of learners* (stimulating cognitive and meta-cognitive activities), and
- *Effective and learner-oriented classroom management* (fostering self-regulation, fostering communication and cooperation among students, learner-friendly environment, clear structure of lessons and effective use of time).

According to Blum, "taking into account (not necessarily all but) certain non-trivial combinations of these criteria will — other conditions being stable — result in better learning outcomes" (p. 2). The criteria have been developed at school level but may be considered, in relevant aspects at least at face value level, also when discussing quality of lectures in undergraduate mathematics.

In the literature *quality teaching* at primary and secondary school levels (for an overview, see Bradley, Sampson, and Royal, 2006) has often been discussed in terms of *quality teacher* — what is a good teacher? This approach may lead to the identification of a list of personal characteristics, either in terms of results from student surveys or from research reviews. As an example of the first perspective, Bradley et al. (ibid.) conclude in a study of secondary students' views on good teaching, that teacher knowledge of mathematics was valued as "the most important quality of the best mathematics teachers," and that "quality instruction is viewed within the spectrum of teachers' caring about the students" (p. 21). In relation to the second perspective, competence oriented approaches tend to parallel teacher proficiency with characteristics of the objectives of student learning (Kilpatrick, Swafford, & Findell, 2001; Niss, 2004). Another way to identify a quality teacher has been to focus on qualification in terms of academic level and professional content of the formal teacher education (Lewis et al., 1999). However, the perspective chosen here is on qualities of a lecture that can be observed and discussed and not on what is required from the lecturer, in terms of competencies or personal characteristics, to give those qualities to a lecture.

Research on Undergraduate Mathematics Lectures

Experience based advice for 'good' lecturing in mathematics is found for example in Krantz (1999), and for lecturing in general in Biggs (2003), but the educational value of large group lectures has often been questioned, in general as well as in mathematics, for reasons such as the following:

- lectures turn the students into passive listeners instead of active learners (Fritze & Nordkvelle, 2003); students' attention cannot be maintained during a whole lecture (Bligh, 1972);
- lectures are most often linearly well ordered outlines of a ready made mathematical theory, not offering a view of mathematics as a human social activity, coloured by creativity, struggles, and other emotional aspects involved in mathematical activity (Alsina, 2001; Weber, 2004),

thus showing only what sometimes is called the "front" of mathematics and hiding the "back";

- lectures are often not understood by the students (Rodd, 2003, p. 15); students do not learn much from traditional lectures (Leron & Dubinsky, 1995); lectures are not effective in stimulating higher-order thinking (Bligh, 1972).

Other critical aspects of the lecture format in university teaching are discussed in Bligh (1972), such as the lack of feedback and social interaction. However, despite the many critical issues raised from research results on lecturing, "the lecture survives, probably because it serves many functions not so well observed in the present research" (Fritze & Nordkvelle, 2003, p. 328). In line with this comment, Rodd (2003) makes the case that "university mathematics departments recognise the potential of lectures, not as information-delivery venues, but as a place where the 'awe and wonder' of mathematics can be experienced" (p. 20), claiming that 'active participation' and 'identity and community' can also be experienced as a 'witness', such as in the context of experiencing in a theatre. Imagination being an essential part of the mathematical experience, effects of inspiration may be an essential outcome from a good lecture.

In this connection it is also relevant to mention the factor of the lecturer as a person, and of humour, both of which have been seen as critical for how lectures are appreciated (Fritze & Nordkvelle, 2003). These are both related to the notion of *teacher immediacy* (Frymier, 1994), referring to the more delicate issues of closeness in classroom student-teacher interaction. Arguments for the importance of personalisation in a mathematics context can also be found in Forgasz and Swedosh, (1997) and in Anthony, Hubbard, and Swedosh (2000). In the study by Anthony (1997) there was evidence that students also place importance on a personal approach by the mathematics lecturer (Anthony et al., 2000, pp. 249-250).

The issue of inspiration is also emphasised by Alsina (2001), who "unmasks" a number of myths about undergraduate mathematics education, which "have a negative influence /.../ on the quality of mathematics teaching" (p. 3), such as self-made teachers, context-free universal content, deductive top-down perfect theory presentation, and non-emotional audience (pp. 3-6).

From the results of a case study, Weber (2004) balanced such negative views of the DTP-format of teaching. He identified three teaching styles used in undergraduate mathematics (small group) lecturing. In the *logico-structural* style a strictly formal way of working was used, with no discussion of the meaning of terms under study. Within the *procedural* lecture style the focus was only on how to fill up the technical details to complete a proof task, while the semantic style aimed to highlight the intuitive meanings of the concepts involved, for example by using diagrams to support the argumentation. In the case studied by Weber, there was a deliberate progression within these lecture styles (in the order presented here), to provide a solid basis for students' understanding.

In a case study, Barnard and Morgan (1996) investigated the match/mismatch between the aims and the practice of a lecturer, analysing one

lecture on "Basic pure mathematics" for first year student teachers. The lecturer set up aims at a general level and at a 'content-related' level. In his practice, his general aims of moving the students from a computational via a descriptive approach towards a deductive approach to mathematical work, were sometimes forced aside when he engaged with specific content-related levels of knowledge of facts, justification, understanding, and 'culture', putting more emphasis of the first two of these levels. These were also the main foci of the assignments and assessment tasks.

More generally, a factor influencing the planning and performing of an undergraduate mathematics lecture concerns the lecturer's ideas and reflections about the aims of the lecture, in terms of beliefs about mathematics and doing and learning mathematics, of his/her students' struggles and ways of conceptualising mathematical ideas and methods. Researching the thinking of undergraduate mathematics teachers, Nardi, Jaworski and Hegedus (2005) identified a spectrum of *pedagogical awareness*, including four levels labelled as *naive and dismissive*, *intuitive and questioning*, *reflective and analytic*, and *confident and articulate* (p. 293). Even if the empirical data were drawn from tutoring, the authors "see teachers' awareness developing in this context as feeding into other, more widespread teaching formats" (p. 293). It seems reasonable to expect that a higher level of pedagogical awareness may contribute to the quality of a lecture from an educational point of view. In the context of limits of functions, this is indeed of relevance, due to the well researched problems students have bringing together intuitive and formal conceptions into a functional understanding (Harel & Trgalova, 1996, pp. 682-686) and the issue of the many different concept images they construct (Przenioslo, 2004).

In a study on different linguistic modes used in an undergraduate mathematics lecture, Wood and Smith (2004) noted that "[l]ecturing is a mixed mode activity" (p. 3), using verbal and non-verbal means to organise students' attention to "written language, mathematical notations, visual diagrams" (p. 3). Wood and Smith observed differences between the lecturer's language in the writing during the lecture, which is constructed dialogically while talking, and the writing in the textbook and computer help files on the same topic, where the latter is more impersonalised. In addition, "in the spoken text... the lecturer makes use of a range of words like *actually*, *fairly*, *obviously* to personalise and introduce values and judgments into the presentation" (p. 7). These differences of modes and representational forms require a lot from the students, and Wood and Smith conclude that "[s]tudent answers to the examination question reveal that there is considerable difficulty in telling a coherent story incorporating the rules of grammar and the use of mathematical language and conventions" (p. 11).

Anthony (1997) investigated factors that influence students' success or failure in first year undergraduate mathematics courses. Data showed that the importance given to lectures was higher by students than by lecturers, both for success and failure, including "boring presentations of lectures" and "non attendance of lectures." Students also "placed more importance than lecturers on active learning and note-taking" during lectures. Comparing student responses,

successful students found “the availability of worked examples in lectures and tutorials” and “clear presentation of lectures” more important than did non-successful students (pp. 60-61). Moreover, the study by Hubbard (1997) showed that students value the information about what is “important” provided by lectures but are often dissatisfied with the format of a lecture as well as lecturers’ ability to teach (for more references on this issue, see Anthony et al., 2000, p. 249). This may well be due to a discrepancy in beliefs and perceptions about the role of lectures between lecturers and students (ibid., p. 250).

A Case Study

The aim of this paper is to explore the notion of quality in undergraduate mathematics lectures, as a basis to reflect on the research question: *What are the main factors that account for quality in an undergraduate mathematics lecture?* Because of the explorative nature of this endeavour, and giving consideration to the points raised in the previous sections, the investigation cannot be based on a predefined general notion of quality. Instead, within the scope of the present study, what accounts for quality in a mathematics lecture can only emerge operationally from the description provided by the provisional answer to the research question. As a consequence, the relevance and usefulness of this characterisation rely on the course of evidence and supporting argument provided by the study.

In order to examine the notion of quality lecture, and to investigate the relevance of the theoretical terms used above for discussing quality lecturing, a case study was performed at a Swedish university, where a lecture in first year calculus in a regular education programme in engineering was observed and analysed. In addition, the lecturer was interviewed in connection with the lecture. These data, in conjunction with the literature review, form the basis of a discussion of the content and usefulness of the concept of quality lecture, as formulated in the research question. Based on the assumption that the lecturer is aiming for quality in lecturing, observations of what the lecturer is doing and how, and interview data about the rationale for these choices, will inform aspects of quality in relation to the lecture.

Method

The mathematics lecture in a large lecture hall is a substantial component of beginning calculus courses in many university engineering and science programmes. This means that these lectures are directed towards large student populations and thus have a major impact on mathematics education at the tertiary level. For the purpose of the study reported here, it was therefore an obvious choice to observe a lecture within this context. Among possible topic areas within a calculus course, limits of functions is one area where student difficulties have been reported (Harel & Trgalova, 1996). Based on these considerations, a case study was undertaken with one lecturer running a calculus course for engineering students. The strategy for gathering data consisted of an

with observation of one lecture about limits of functions and a follow up interview with the lecturer. The male lecturer is well experienced in lecturing, is a professionally trained mathematician, and has co-authored a textbook in calculus. This means that the case represents a common situation regarding the phenomenon under study and thus may provide viable data for exploring the issue of a quality lecture. However, the results from one case cannot form the basis of a generalisation to the whole class of undergraduate mathematics lectures but only as an input for the construction of a first tentative model of lecture quality, the viability of which must be questioned by further studies, from the perspectives of students, lecturers, and educational goals.

All data were collected by the author, who attended the lecture and took extensive field notes, including a full "blueprint" of all that was written on the whiteboard by the lecturer, short hands of the words the lecturer used to comment what he wrote, as much as possible with quotes of sentences or phrases that the observer found relevant for the study, and finally notes about observable behaviours such as the use of gestures. A video recording was avoided due to the risk of influencing the lecturer's performance. Immediately after the lecture the field notes were transcribed to the format of the lecture protocol as presented below.

Two days before the interview with the lecturer, which lasted for about 45 minutes in an informal setting, the lecturer was given the transcribed protocol (in Swedish). A sheet with nine questions formed the basis of the interview. It was given again to the interviewee at the commencement of the interview. These questions were based on the lecture protocol and issues noted in the literature review. In order to have a relaxed and informal discussion, there was no audio recording made during the interview but extensive notes were taken by the interviewer (the author) and transcribed immediately after the interview. The exact words of the interviewee were written down as much as possible. The summary of the interview protocol shown below contains the main issues raised, by using quotations (in italics) and summaries put into a story-like format, structured by the interview questions. A short time after the interview the summary was shown to the lecturer who confirmed that it gave an accurate account for what he had said and meant. To give the reader insight into what actually happened, and at least partly why, these two protocols of empirical data are displayed as narratives, as fully as space allows.

A Conceptual Framework

The data will be analysed within a *conceptual framework* (Lester, 2005). Instead of relying on one particular theory, a conceptual framework is "built from an array of current and possibly far-ranging sources", and can be "based on different theories and various aspects of practitioner knowledge, depending on what the researcher can argue will be relevant and important to address about a research problem" (Lester, 2005, p. 460). The validity for the chosen framework is context dependent, which is its strength considering the implications of the research. This method is found to be relevant and useful in an explorative study with the present aim to identify critical aspects of a lecture that may account for its quality.

The framework will be partially based on the analytic categories, notions, and results that emerged from the literature reviewed in the previous sections. To account for the structure and organisation of the lecture, the notion of lecturing styles (Saroyan & Snell, 1997; Weber, 2004) will be applied. The critical aspect of the lecturer as a person will be discussed in terms of personalisation and teacher immediacy (Anthony et al., 2000; Frymier, 1994), and inspiration and "awe and wonder" (Rodd, 2003). Related to both the way of lecturing and personalisation is the mixed-mode character of lectures (Wood & Smith, 2004), with the use of metaphors (Lakoff & Núñez, 2000) and gestures as important *semiotic means of objectification of knowledge* (Radford, 2003), supported by the lecture format.

For analysing epistemological aspects of the mathematical knowledge displayed in the lecture, I will use the theoretical construct of *didactic transposition*, and the organisation of that knowledge in terms of *tasks, techniques, technology*, and *theory*, incorportated within anthropological theory of didactics (ATD) as outlined in Barbé, Bosch, Espinoza, and Gascon (2005). What kinds of tasks are discussed during the lecture and what methods/techniques are used to solve these tasks? How are the techniques used explicitly related to the theoretical tools (the technology) and theories that explain and justify their use? In the ATD, practical knowledge (tasks and techniques) and theoretical knowledge (technology and theory) together constitute a unit of analysis referred to as a mathematical organisation, shaped by the didactic transposition and institutional constraints. Related to this, one of the roles of a lecturer is to identify what is important in the course from a mathematical point of view, and what counts as knowledge in mathematics — that is the process of *institutionalisation* of knowledge. One aspect of this process is the establishment of *socio-mathematical norms* (Yackel & Cobb, 1996).

In addition, as a teaching format, *general criteria* for quality teaching (Biggs, 2003; Blum, 2004) also need to be considered for lectures of the kind in focus here.

The Lecture Protocol

The lecture takes place in a first year calculus course for engineering students. There are around 140 students in the tiered lecture hall. Behind the podium there are three sets of three vertically adjustable whiteboards. The lecture takes place at 10.15 to 12.00, with a 15 minutes break at 11.00. The topic of the lecture is standard limits, and is the third lecture of the course following on from an introduction of the concept and discussion of basic properties of limits and of continuous functions.

The protocol (see Figure 1) is structured in three columns: to the left is a "blueprint" of all that was written on the whiteboard, in the middle some of the words spoken by the lecturer, and to the right the author's comments for clarification. Vertically, time is running chronologically. Some reflections on smaller sections of the protocol will be provided.

Without any informal introduction, the lecturer begins by simply stating facts, telling what there is. He then goes on to say that we shall look at some different kinds of functions and standard limits. A motivation is given by saying that they will be used when finding the derivative.

	We continue with limits.	The very first words said.
When $x \to \infty$ we have	*To be able to compare different kinds of functions we will need standard limits.*	Gives general comments.
(1) $\dfrac{\text{In } x}{x^{\alpha}} \to 0$, with $\alpha > 0$ (α constant)		
(2) $\dfrac{x^{\alpha}}{\alpha^{x}} \to 0$, with $\alpha > 1$ (α constant)		Gives examples orally.
In x, x^{α}, α^{x} are ordered by size (for large x)	*Speed table.*	

Figure 1. Protocol 1.

After listing the standard limits, thus defining which are important (institutionalisation), the lecturer goes on to prove "some of them" (see Figure 2). In fact, he proves all of them but one, for which he gives a hint as to how to tackle it. The mathematical knowledge set out in this opening of the lecture is organised

When $x \to 0$ we have	*We shall look at some of the proofs.*	
(3) $\dfrac{\sin x}{x} \to 1$ (radians)		
(4) $\dfrac{\text{In } (1+x)}{x} \to 1$ (as we have already shown)		
(5) $(1+x)^{1/x} \to e$		
(6) $\dfrac{e^{x}-1}{x} \to 1$		Referring to previous lectures, where limits of the "kinds" $0 \cdot \infty$, $\dfrac{0}{0}$ and $\dfrac{\infty}{\infty}$ were identified as "difficult."
(7) $x^{\alpha}\,\text{In x} \to 0$ with $\alpha > 0$		
Proofs of some	*They are all of the difficult kind.*	

Figure 2. Protocol 2.

around the presentation of the technology before the presentation of the tasks for which the technology has been developed to solve. The rationale offered by the lecturer at this stage is a within mathematics argument that the standard limits will be needed later, for the derivative, a concept that has not yet been discussed in the course. Staying at a theoretical level of the mathematical organisation, the lecture continues in a procedural style.

The lecturer takes it for granted that the students are aware of why it is a good strategy to "squeeze" (See Figure 3). There is no explicit reason given why to "look at the square root of x." There is no comment about which aspect the obtained inequality is considered "bad" or "better." How this is to be understood by the students is unclear. The formal cogency of the procedural lecture style is mixed up with everyday expressions like "this lump." A meta-comment is inserted about how it is sometimes possible to show a general case from a special case. The example just shown illustrates the point made in the comment.

Proofs of some		
We start by showing that $\frac{\ln x}{x} \to 0$ as $x \to \infty$	*We want to "squeeze".*	Pointing here and there in the expression.
As known we have $0 < \ln x < x–1$, for $x > 1$	*We could look at the square root of x.*	
From this it follows that $0 < \ln\sqrt{x} < \sqrt{x}– 1$, for $x > 1$ i.e. $0 < \ln x < 2\,(\sqrt{x}–1)$. Whence we have	*A bad but better inequality for large x.*	Using log-rule for In $\sqrt{x} = \frac{1}{2}$ In x
$0 < \frac{\ln x}{x} < \frac{2\,(\sqrt{x}–1)}{x} = \frac{2}{\sqrt{x}} - \frac{2}{x}$	*Squeezed.*	
$\to 0$ as $x \to \infty$ $\quad\quad$ $\to 0$ as $x \to \infty$ $\frac{\ln x}{x} \to 0,\ x \to \infty$		Concluding.
The general case: $\frac{\ln x}{x^{\alpha}} = \frac{\frac{1}{\alpha}\ln x^{\alpha}}{x^{\alpha}} \to 0$ as $x \to \infty$	*This lump tends to zero.*	Marks in the second fraction all but $\frac{1}{\alpha}$.
$y = x^{\alpha} \to \infty$ for $\alpha > 0$, $x \to \infty$	*Not so uncommon in math that you start with a special case.*	Comments on the method.

Figure 3. Protocol 3.

The prevalent formal rigour here is complemented by a reasoning supported by a diagram (see Figure 4) which is not logically valid (the critical stance is $x < \tan x$), something that is seen also in the use of an everyday metaphor in which the lecturer is making choices on behalf of students between two pathways. For example, when investigating an even function, students are left to fill in the

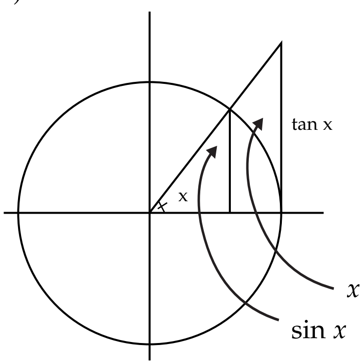
(2) $\frac{x^{\alpha}}{\alpha^{x}}$ Let $\alpha^{x} = y$, i.e. $x = \frac{\ln y}{\ln \alpha}$, $y \to \infty$ $(\alpha > 1)$ (exercise) (1)	*If you do this you will get back that first standard limit.*	
(3) 	*Then I want to make a diagram. We can start with small x.*	Pointing, making gestures.
For $0 < x < \frac{\pi}{2}$ $\sin x < x < \tan x$	*If you would want to run along this path or that path I claim you would choose this one.*	Pointing in the diagram that the arc x and the vertical sin x are about the same for small x, and that tan x is longer.
Divide by $\sin x$ (> 0) to get $1 < \frac{x}{\sin x} < \frac{x}{\cos x}$, i.e. $\cos x < \frac{\sin x}{x} < 1$ $\therefore \frac{\sin x}{x} \to 1$ as $x \to 0+$ We also have $f(x) = \frac{\sin x}{x}$ even, i.e. $f(-x) = f(x)$ so $\lim_{x \to 0-} \frac{\sin x}{x} = 1$ $\therefore \frac{\sin x}{x} \to 1$ as $x \to 0$		The lecturer is using gestures to illustrate coming from the right or the left: his left arm is out and his right hand in front moving from right to left, and vice versa.

Figure 4. Protocol 4.

details themselves about how this property is used here. Another intuitive trait of this *mixed mode* presentation is gesture. To say "tends to" and to use arrow notation is another way of displaying an intuitive conception of limits. The *lim* symbol is used once, for the first time during the lecture, but quickly abandoned. However, it appears soon again, maybe for practical notational reasons.

The lecturer's words (Figure 5) along with (5) give emotional and independent life to the mathematical objects, but they hide the use of the continuity of the exponential function. After finishing the proofs, some "connecting" comments are made about how this knowledge can be used. It

(4) proved previously (5) $\lim_{x\to 0}(1+x)^{1/x}=\lim_{x\to 0} e^{\ln(1+x)^{1/x}}=\lim_{x\to 0} e^{\frac{1}{x}\ln(1+x)}=e^1=e$ $\to 1$, by (4)	*It is not nice when both the base and the exponent are moving. ...use log-rules, taking down the exponent*	Refers to the definition for rewriting the logarithm. Marks $\frac{1}{x}\ln(1+x)$ using the words *that lump.*
(6) $\lim_{x\to 0}\frac{e^x-1}{x}$ =/let $e^x-1=y$/ = exercise (7) $\lim_{x\to 0} x^{\alpha}\ln x$ =/let $y=\frac{1}{x}$, $y\to\infty$ since $x\to 0+$/	*Those are the ones we have, previously extracting from polynomials /.../ later on Maclaurin expansion /.../ polar and some crap, avoid standard limits, but building on /.../ Now we shall look at some examples.*	He is here talking about the inverse, the logarithm. One can hear, immediately after the last sentence, the sounds of note books being opened.

Figure 5. Protocol 5.

must also be noted that many students are only listening and not taking notes until the lecturer announces: "We now shall look at some examples," when at once there is a sudden loud sound of note books being opened.

From now on the mathematics presented is organised through tasks, and the techniques to be used in solving these tasks (see Figure 6). Focus is on practical knowledge, the 'know-how'. The lecturer is in example 1 setting up socio-mathematical norms for how one "should" do, at the same time as he is building on an intuitive grasp of the variable concept as general-exchangeable. Again, teacher immediacy is strong through his use of humour. One could call it a *pseudo formalism* to say only "upside down" about the line $\lim_{x\to 0}\frac{y}{\sin y}=\frac{1}{1}=1$. It can also

ex 1) $\lim_{x\to 0}\frac{e^{\sin 7x}-1}{x}=\lim_{x\to 0}\frac{e^{\sin 7x}-1}{\sin 7x}\cdot\frac{\sin 7x}{7x}\cdot 7=1\cdot 1\cdot 7=7$ $\frac{\sin\triangle}{\triangle}\to 1$ as $\triangle\to 0$ $\frac{e^{\triangle}-1}{\triangle}\to 1$ as $\triangle\to 0$ (it must be the same expression in $\triangle$)	*Not true — also not true — now it is true* *This is how you should read standard limits.*	The lecturer adds on factors one by one, students laugh. The lecturer talks and points, from close and from far away, points at 1·1·7.
ex 2) $\lim_{x\to 0}\frac{\arcsin x}{7x}=$/let $y=\arcsin x$, $x=\sin y$, $y\to 0$ as $x\to 0$/ $=\lim_{x\to 0}\frac{y}{\sin y}=\frac{1}{1}=1$ NOTE: $\frac{\arcsin x}{7x}\neq\frac{\sin(\arcsin x)}{\sin x}=\frac{x}{\sin x}$ $2=\frac{\pi}{\pi/2}=\frac{\sin\pi}{\sin\pi/2}=\frac{0}{1}=0,\ 2=0$	*We do one more. You get $\frac{0}{0}$.* *Trying to get rid of* arcsin x. *Upside down. Try some angles.* *Now we will have a break.*	Short explanation. Time is exactly 11.00

Figure 6. Protocol 6.

be noted that the lecturer is trying to prevent students from making a common mistake (cancelling "sin" from the numerator and denominator, as if it was a number that can be divided).

The examples somehow speak for themselves. No motivation is given as to why these particular tasks were chosen (see Figure 7), or why the rewriting technique using log-rules is applied. The presentation turns into a kind of ritual with its *raison d'être* taken for granted, as it seems, by lecturer and students alike. All the examples are lined up according to a "this is how to do it" model of presentation.

ex 3) $\lim\limits_{x\to\infty} x(\ln(1+x) - \ln x) =$ $= \lim\limits_{x\to\infty} x(\ln(1+\frac{1}{x}) = \lim\limits_{x\to\infty} \dfrac{\ln(1+\frac{1}{x})}{\frac{1}{x}} = 1$ (stl, since $\frac{1}{x} \to 0$ as $x \to \infty$)	*We do one more.* *We take log-rules.*	The first and second wordings only, as introduction to the second session.

Figure 7. Protocol 7.

The formulation of the next task, example 4, is written down without any comments:

Investigate $f(x) = x\sqrt{\dfrac{x}{x-1}}$ for large x ($x \to \infty$)

In a careful procedural style it is demonstrated how the function tends to infinity and can be approximated by a linear asymptote, including the standard techniques to find the equation of that straight line. After this purely algebraic exposition, the lecturer concludes with a diagram (see Figure 8).

The main part of the second half of the lecture is thus spent on techniques to solve different types of tasks (practical knowledge), using the technology introduced during the first part of the lecture. Only a minor part of the lesson focuses on the theoretical superstructure or validation of the techniques (theoretical knowledge). The character of this mathematical work is mainly algebraic and non-numerical, and includes some instances of imagining. The comments of the lecturer contribute to the creation of socio-mathematical norms. For example, he notes that a diagram can illustrate an idea without being exact. How the students understand this is not clear — what reason could there be to draw an incorrect diagram? Gestures used along with choices of words create patterns for thinking, which might well constrain conceptualisations in certain standard situations.

In a final example the convergence of a number sequence is shown:

$$\alpha_n = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \ldots + \frac{1}{n+n},\ n = 1, 2, 3, \ldots$$

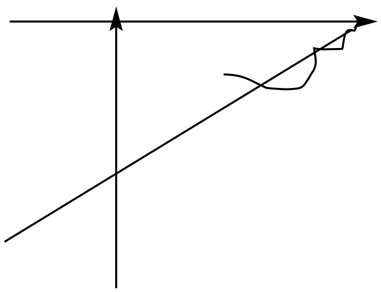
$\therefore\ y = x + \frac{1}{2}$ is an asymptote to f as $x \to \infty$ (check the asymptote as $x \to -\infty$) $y = x + \frac{1}{2}$ 	*It could look like this but it doesn't.* *The calculation does not show how big is the difference.*	Points out that f can also have a vertical asymptote, moving his right arm vertically to illustrate his point. Draws a sketch. Laughter. Gives example, laughter.

Figure 8. Protocol 8.

When working on this sequence, again applying the 'squeezing' technique, some meta-level comments are made, diagrams are drawn and certain gestures are used by the lecturer.

Summary of Interview Protocol

The interview starts by talking about the role of the lectures within the course. The lecturer says he is

> trying to "extract" the main core of the course, some parts more informally, the details can be found in the literature. To present some things carefully, show how to do rigorous proofs for those who are interested, knowing that students do not always read the textbook. Pointing at critical details, sometimes giving examples.

For the definition of a limit of a function he used in his first lecture the formal ε — δ characterisation to some extent, but put more emphasis on an intuitive image of closeness, using diagrams of graphs. The reasons for doing limits are driven by mathematics itself — derivatives, integrals and asymptotic behaviour. These latter concepts are motivated by applications outside of mathematics.

> I emphasise that limits is the most important concept for the whole course, all other concepts are building on it.

The lecturer maintains that lectures in this course state that lectures are cheaper than other forms of teaching, and that when doing formal lectures there is less

lecturing during classes/tutorials. The lecture format is good, and students most often attend lectures to a greater extent than classes.

He finds it difficult to describe what kinds of (specific) goals he sets for each lecture.

> I want to present, to make things seem true, the most important I think is that students believe they understand better what a concept means. To exemplify what you can handle practically, to illustrate the standard way of doing things. Lectures can look very different, some being richer with examples.

Later on in the interview, discussing quality, he repeats this argument:

> Some lectures get a little colourless, providing a base to build on, lectures differ.

When asked what the students got from this particular lecture or why students attend lectures, he says that they normally go to the lectures,

> I don't know why. It is an easy way to get something done, they think they can use things from the lecture, collect materials, thinking the lecturer will say something that is useful for the exam.

Even if he says he is not good at grasping whether or not students understand, it sometimes happens that he experiences a "sense" that something is difficult, and then makes some modifications to his plan.

We discuss what makes a lecture a good lecture, the issue of quality. He returns to arguments similar to what he said about goals:

> I can sometimes feel it has been good, sometimes experience fears - not interesting enough, too many examples? But the students maybe see it differently, that it is good with many examples. The students should get some (beginning of an) insight, a better understanding of some concepts, a better image, make them believe that some things may hold. There is no need to include everything in detail, just do some more "popular" descriptions and let the students themselves fill in the details from the textbook. At least some of them do this.

He goes on to describe how the students should experience some kind of engagement, and get some kind of deeper understanding:

> One has to make the basic standard limits one's own. I think it is easier to remember something that you once have understood why it is true. Getting the basic picture, you remember it whether you want it or not. That is how I felt when I was a student.

He points out that sometimes there are things that are more difficult to grasp this way, such as limits involving the logarithm.

> If this lecture was of high quality I don't know, maybe somewhat dispersed. Other lectures can be more coherent. [That this course is a requisite for several study programmes, it] makes you less free with, for example, the order in which things are presented.

He notes that students often come asking questions during the break or after the lecture. It can be about explaining things on the whiteboard, to fill in some details, but it can also be what their teacher at high school has said, something from applications or about what will happen later on in the course. It is the more able students who ask these questions.

When commenting on this particular protocol, he says:

> I was not always careful with the wordings, such as the difference between a function and its graph. The protocol gives an impression that all was kind of relaxed, which is something I strive for.

Concerning the lack of numerical interpretation of limits, he gives the reason that:

> It is easy to get a misleading impression from the numerical behaviour of for example $x^{\frac{1}{2}} \ln x$ as $x \to 0+$. I don't know if the students would get an easier access to the concepts this way.

When asked about learning to lecture he says that it is learning by experience:

> I do things slower now than some years ago, when I wanted to cover all topics, now I skip some and leave it to the students.

It is not common to visit others' lectures but there are many informal discussions with colleagues.

Discussion

The first thing to note is that questions about lectures such as those in focus here, (provided it is a lecture as part of a course), cannot be treated in full isolation from the other formats of teaching that constitute the course. However, there may be some quality characteristics of a general kind that apply to lectures as such. After a few lectures the "crowd" (the students) get used to the lecturer's way of lecturing. The students who attend the lecture do so, it can be assumed, to increase their chances of passing the course — something also expressed by the lecturer in the interview. This implies that the lecturer has the advantage that the "crowd" is not only willing but often even anxious to listen and take notes. The protocols support these observations, even if student interviews, which were not done on this occasion, would be needed to substantiate this claim. What the lecturer puts forward is considered (by the students) the core of the course — the most important aspects (Hubbard, 1997), thus establishing the institutionalisation of knowledge. This can also be inferred from the common tradition among students to copy and even sell lecture notes, even in cases where there is a textbook available. Another feature of this kind of lecture is that the lecturer has a fair control of the listeners' (at least formal) pre-knowledge related to the content of the lecture. Viewed from these considerations, the perceived quality of the lecture is relative to the expectations of the students.

In project tasks, organised group work, and steered individual tasks, the role of the teacher is less directly visible to the students. The personalisation of

teaching is reduced, as is the social and affective interplay between students and teacher. These aspects of the teaching situation have an influence on the process in more dialogical classroom management, and make the teacher as a *Person* critical to a higher degree. This is even more the case in a lecturing format, especially in a lecture hall situation with an audience of a large "crowd" of students. In the lecture and interview protocols the importance of this issue of teacher immediacy was present, thus constituting a quality aspect.

Of the three lecture styles identified by Saroyan and Snell (1997), the lecture observed here must be classified as content-driven. Selected mathematical content is presented in 'splendid isolation' to the students without their visible active intervention into the teaching process. There is also no summary provided, or elaborated links to other parts of the course. Considering the teaching styles observed by Weber (2004), this lecture displayed a mixed mode, with no visible progression through the different styles. Regarding quality, it was made clear in the interview that the lecturer is trying to fulfil the two aims of making the students understand (using a *semantic* style) and to illustrate the standard ways of doing things (using a *procedural* style). In his practice these two aims were not separated.

Related to the content aspects, the organisation of the mathematical knowledge was formally separated in a theoretical part (the *technology*) during the first half of the lecture, and a practical part (the *tasks* and *techniques*) constituting the second half of the lecture. In terms of the ATD, the mathematical knowledge in focus during the first part of the lecture was displayed as a ready-made technology useful for solving problems not known for the students. The *raison d'être* of mathematics and its theoretical techniques were taken for granted, and the students were not invited to take an active part in the reasoning process. However, the list of standard limits could also be viewed as tasks in themselves, to be 'solved' by formally proving them, using general theoretical techniques such as formal estimations by inequalities, substitutions, and algebraic manipulations. During the second 'know-how' part of the lecture, the tasks chosen were all within the realm of pure mathematics, solved by applying the technology of standard limits through algebraic treatments of the given expressions. The mathematical organisation was thus characterised conceptually by a focus on the algebra of limits rather than on its topology (Barbé et al., 2005). Considering the quality of the lecture, this focus on the technique to some extent contrasts with the aims for student understanding, expressed by the lecturer in the interview. Seen as a whole, the lecture defines a coherent mathematical organisation, shaped by the institutional tradition for this kind of course, and expressed in the textbook used. It is the result of the didactic transposition of the target knowledge. For the lecturer, the balance between theory and tasks (examples) seems to be a quality feature from the students' point of view. He also alludes to institutional constraints as delimiting his freedom to structure the lectures.

Quality Aspects of Lectures

Referring to the observations of the lecture, ten quality aspects of lectures that emerged will here be further discussed. Students attending lectures are confronted with new knowledge, and this makes the issue of information delivery critical for the perceived quality of the lecture. There are also issues to do with connections, socio-mathematical norms, and mathematical mind. To account for the quality of how these issues were treated requires attention to rigour-intuition, algebraic-imagistic modes, gestures, inspiration, and personalisation. Finally, the general quality criteria need to be taken into account.

Information delivery. The lecture is rich in presenting mathematical information such as basic theorems, proof methods, typical tasks and problem solving techniques. At a face level of analysis, the lecture is a demonstration of selected facts and procedures. In terms of quality one must ask *why* the topic is presented as a lecture, *what* information is chosen and *how* it is demonstrated. Interview data do not provide a clear reason for why lectures are used but give some insight into the personal process of didactic transposition, by words such as *trying to "extract" the main core of the course, Pointing at critical details*, and *to illustrate the standard way of doing things*. The format, within a content-driven and semantic-procedural mixed mode teaching style, may be characterised mainly as traditional DTP, with the D-part already mostly undertaken in a previous lecture. During the second half of the lecture, focus was solely on the application of the theorems proved to specific problems on limits, demonstrating useful techniques. A more appropriate description of the lecture would therefore be *theorem — proof — application*, that is a TPA format. However, the A did not include modelling discussions but still only displaying the 'front' of mathematics. Concerning the how issue, it will be discussed further below.

Connections. In this lecture no external connections (outside mathematics) are made. The interview made clear that some applications are at least mentioned in other lectures — in fact the lecturer states that lectures can be very different. After proving the standard limits some internal connections (within mathematics) are given. They are also made at the very end of the lecture. On the whole, however, these connections are sparse and never worked out in detail, apart from a referenece to a known inequality in the proof of standard limit (1). Under this heading the level of *coherence* of the lecture can also be included, in relation to how well the different parts of the lecture are connected. This is seen as a condition for quality according to the interview. As mentioned above, the lecture had two distinct parts, a theoretical part and an "applied" part, that is a demonstration of *related* techniques on typical tasks, the "know-how" connected to the theoretical tools (the technology). From a quality point of view, these connections were based on deductive principles rather than didactical, and took into consideration motivational aspects to justify the introduction of the technology.

Rigour-intuition. Throughout the lecture, through the proofs and solutions of examples, mathematical rigour is maintained. Only on one occasion, discussing

the inequality $x < \tan x$, is rigour replaced by intuitive reasoning, or rather an attempt to convince, based on a diagram. This balance of rigour-intuition contrasts with what the lecturer says in the interview, that the most important is to get the students get a feeling of understanding: *There is no need to include everything in detail*. At the same time he wants to *present some things carefully, show how to do rigorous proofs for those who are interested*. In this lecture most things he presented were done so carefully, supplemented by rhetoric and gestures in keeping with a more intuitive approach. This effect was also present in the study by Wood and Smith (2004) and can be seen as a quality aspect. There are also situations where things are taken for granted. These could possibly present a problem for some of the students, such as algebraic rearrangements or taking an equality or theorem as known.

Algebraic-imagistic modes. The different mathematical registers of algebra and diagrams are closely linked to the previous aspects of rigour-intuition. The algebraic mode dominates, while diagrams are presented only on four separate occasions. Each of these diagrams functions as an aid to reasoning, and on one occasion is the only basis for a logical conclusion. This is the occasion when the path metaphor (Lakoff & Núñez, 2000) was used. The deliberate choice of defining a mathematical organisation based on the algebra of limits rather than its topology (based on the meaning of the limit concept, using imagistic and numerical reasoning), affects the quality of the lecture.

Gestures. As seen from the lecture protocol, gestures are often used by the lecturer to make ideas visible — to illustrate. In the interview the lecturer puts an emphasis on the word *illustrate* as an overall goal for his lecturing, thus giving it a quality status. This aspect can be seen as part of the game of lecture, as features of acting. In addition, illustrating adds non-symbolic and non-discursive elements to the semiotic objectification of knowledge, which can function as critical elements in the meaning-making processes of the students.

Socio-mathematical norms. The written mathematical messages on the whiteboard play the role of institutionalisation, stating what officially counts in mathematics. Similarly, the oral messages such as *This is how you should read standard limits*, tell the students "how to do it", These features, taken together, establish the socio-mathematical norms for the lecture group, and are possibly viewed by the students as necessary for passing the exam. The sudden activity of writing in the notebooks when examples were announced, supports this claim, as do the interview responses: Students may think that *the lecturer will say something that is useful for the exam*.

Mathematical mind (ways of doing/thinking, beliefs, and attitudes). In carrying out the proofs and the examples, the lecturer is also conveying ideas about thinking and useful techniques in mathematics. He is, at the same time, acting like a model mathematician and doing what he is preaching, sometimes simply doing it without giving reasons or excuses (*We do one more or then I want to make a diagram*). As a model, he is a person, using not only the formal language of mathematics but also metaphors and everyday wordings (*We want to "squeeze"*), including normative expressions such as *It is not nice when both the exponent and*

the base are moving. The students can witness "live" how mathematics can be done, as discussed in Rodd (2003). It may well be at the technique level of the mathematical organisation where teacher immediacy has most potential to affect quality.

Inspiration. As observed in the literature review, the issue of inspiration is by many writers seen as one of the key features and potentials of lectures. It is thus an important quality aspect. However, in the interview there is only a brief mention of the fact that students should experience some kind of engagement. The objectives of the lecture mainly concern student conceptual understanding and demonstrating functional mathematical tools. Thus, from the data presented here, one can only infer from the lecture protocol the potential "awe and wonder" that students might experience, along with inspiration to go on doing exercises on their own or in class. That students were in fact active listeners is evidenced by their reactions on different occasions, such as laughter or taking notes. The examples chosen for demonstration were to some extent at an advanced level. The last one (the number sequence only mentioned above) was possibly out of reach for many to understand "in real time." This feature could contribute to inspiration: was it really possible to decide on the convergence of that number sequence, and will we even learn how to compute its limit value later on?

Personalisation. The lecturer as a person is clearly visible in the lecture notes by his use of a personal non-formal language to balance the algebraic flow on the whiteboard, his use of gestures and humour. The interview shows that in his lectures he is striving for a relaxed atmosphere, which can be one way of expressing that he and his students can have a nice time together doing mathematics in a non-authoritarian mode. When this is the case, it accounts for parts of the quality of the lecture. The impression from the observer was that *Mathematics* was the dominating "person" at this lecture but that it was communicated through a *Person* as a human activity, even if this was performed within a "front" TPA format.

General criteria for quality mathematics teaching. In relation to the teaching quality criteria by Blum (2004), the protocol and interview data show clear traits of parts of all three strands (demanding orchestration of the teaching of mathematical subject matter, cognitive activation of learners, and effective and learner-oriented classroom management), such as competence orientation, stimulating cognitive and meta-cognitive activities. These provided a clear structure and effective use of time. Other factors are less visible, partly due to the very format of a lecture. The trait most noticeably absent in the lecture was communication and cooperation among students, a teaching quality emphasised also by Biggs (2003). A possibility to address this issue, as well as the problem of attention (Bligh, 1972), could be to use one or more short sections of the lecture for discussion between pairs of students of some critical topic. The motivational context raised by Biggs (2003) was discussed above in the section on connections.

Conclusions

The discussion above clearly shows the complexity and richness of the different educational aspects of a lecture in undergraduate mathematics that come into play during a period of only 90 minutes. Within a TPA format (theorem-proof-application), this lecture was content-driven and rich in information, used a mixed mode of semantic-procedural teaching styles, exhibited a formal separation of theoretical and practical knowledge combined with an overall strong coherence, was higher in rigour than the aims stated, and displayed a dominance of an algebraic mode over imagery. Although rich in gestures and informal language, at the same time the lecture established socio-mathematical norms for the course and for Mathematics, within a relaxed, apparently collaborative, environment for doing mathematics. It was framed by some general established qualities of mathematics teaching. Constrained by the lecture format the lecturer used various semiotic means to objectify the target knowledge, as he conceived it, *for* the students. Through this, the students were being prepared to work in classes/tutorials elaborating on this knowledge. Only an analysis of the combined effect of the different teaching formats offered to the students can provide a basis for evaluating the full role of the lecture within this educational game. However, each format also has its own rationale, and the purpose of this study is to discuss the quality of a lecture *per se*.

To structure the conclusions, six of the ten aspects discussed previously may be categorised pair-wise into three discipline-related dimensions, namely, mathematical content, mathematical process, and institutionalisation (Figure 9). The way these are exposed and integrated by the lecturer presenting a more or less coherent image of mathematics is, based on the literature and data presented here, one of the main factors accounting for lecture quality. I call this the 'mathematical exposition' of the lecture.

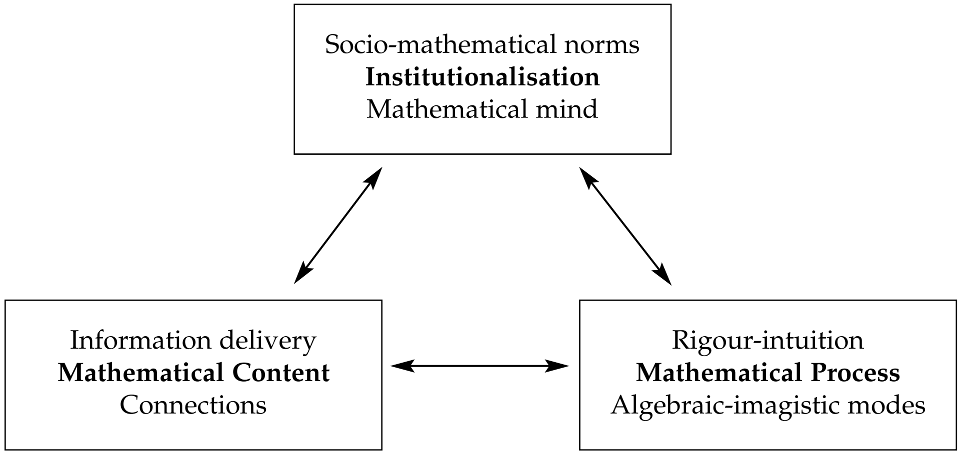


Figure 9. Mathematical exposition.

In this study two additional factors critical for quality in lecturing have emerged, *teacher immediacy* (including personalisation and use of gestures) and *general criteria* for quality teaching. Figure 10 is an attempt to capture the complexity between three main factors that have thus been identified to account for quality in undergraduate mathematics lectures. The arrows are to be interpreted in a systemic way: no one of the three factors can be considered, when discussing lecture quality, without taking the other two into account. I thus hypothesise that high quality within each factor, in conjunction with a strong alignment of their orchestration, are some of the key aspects of a quality lecture. In such cases, the role of the lecture may go beyond a mere information delivery function, and add elements of inspiration and in some cases an experience of 'awe and wonder' in students, with a potential to significantly support learning. High quality in each factor is, among other things, relative to educational objectives and student expectations, complementing a high level of pedagogical awareness, not only among practitioners but also within the educational system.

Is it thus possible to construct a set of criteria to discriminate between lectures in terms of quality from the observation point of view taken here? One option could be to assign an observed "level" to factors such as those discussed above. The kind of validity such a procedure could produce would probably become more well defined by basing the evaluation on whole series of lectures in a range of courses. Indeed, in the interview the lecturer emphasised that lectures even in the same course can be verydifferent. However, as argued above, and due to the crucial role of the lecture format on teacher immediacy, the full relevance of such an approach for developing teaching quality cannot be easily identified. Instead the meaning of the kind of study presented here lies in its potential to initiate a more focused discussion, based on well researched theoretical terms and empirical observations, on what in fact takes place in undergraduate mathematics teaching. Such observations would identify critical factors and would develop among practitioners an increased pedagogical awareness. This could lead to a development of quality in undergraduate mathematics teaching, not only in lecturing but as an integrated educational enterprise.

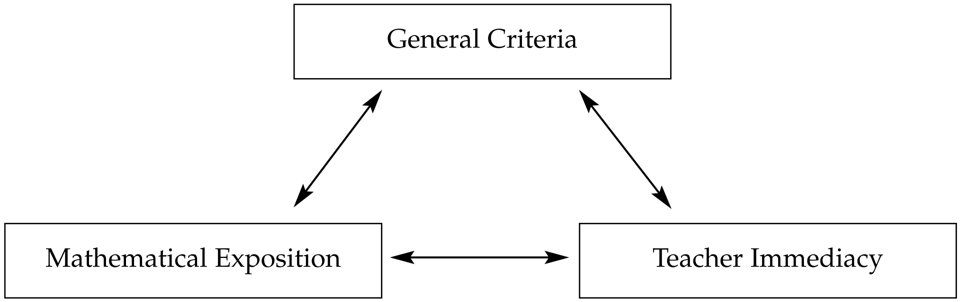


Figure 10. Main factors for lecture quality.

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